reduction in 1/f noise. It should be noted that a backward diode mixer can be operated with an order of magnitude lower local oscillator power than mixer diodes used today. Very satisfactory performance was also obtained using 30-Mc IF and in low-level detection, in spite of the fact that no real attempt was made to optimize the diode fabrication technique. In fact, there are good theoretical reasons for believing that other wafer materials may well prove to be superior to germanium for the applications considered in the paper. Since the backward diodes are virtually independent of the lifetime of minority carriers or of the surface treatment, they can tolerate larger doses of nuclear radiation than conventional mixer diodes. The tunneling portion of the I-V curve is also expected to be substantially independent of temperature.

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Design Theory of Up-Converters for Use as Electronically-Tunable Filters*

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Summary—The up-converters discussed use a single diode, a wide-band impedance matching filter at their signal input, a moderately wide-band impedance matching filter at their pump input, and a narrow-band filter at their sideband output. With a narrow-band filter at the sideband output, the frequency which will be accepted by the amplifier can be controlled by varying the pump frequency. Analysis of the impedance matching problem involved shows that tuning ranges of the order of a half-octave to an octave are possible. Theory is presented for both the lower-sideband and upper-sideband types of tunable up-converters and for the design of the required impedance-matching networks. It is shown that, because of the pump input bandwidth required, it will generally be necessary to accept some mismatch at the pump input. But, by use of a properly designed impedance-matching filter, the reflection loss can be kept nearly constant across the pump band, and the incident pump power required is not unreasonable. It is seen that properly designed devices of this type using voltage-tunable pump oscillators should have wide tuning range, fast tuning capability, a useful amount of gain, no image response, and a low noise figure.

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I. INTRODUCTION

A. Description of the Proposed Devices

PREVIOUS work dealt with the application of filter theory to the design of wide-band parametric amplifiers and up-converters.[1] The present discussion applies a similar theoretical approach to a different but closely related problem. The objective will be to obtain an electronically controlled *wide tuning range* using up-converters having a wide-band input-impedance-matching filter, a narrow-band output-impedance-matching filter, and a voltage-tunable pump oscillator such as a backward-wave oscillator.

Defining f as the input frequency, f' as the sideband output frequency, and f^p as the pump frequency, for a lower-sideband up-converter the output is at the lower-

[1] G. L. Matthaei, "A study of the optimum design of wide-band parametric amplifiers and up-converters," IRE TRANS. ON MICROWAVE THEORY AND TECHNIQUES, vol. MTT-9, pp. 23–28; January, 1961.

sideband frequency

$$f' = f^p - f. \tag{1}$$

For an upper-sideband up-converter, the output is at the upper-sideband frequency

$$f' = f^p + f. \tag{2}$$

For either type of up-converter, tuning action can be achieved if a narrow-band filter is used at the output so that only frequencies equal or very nearly equal to a specific frequency f_0' can be passed. If the pump frequency f^p is varied, then the input frequency which will be accepted by the amplifier will be given by

$$f = f^p - f_0' \tag{3}$$

for the case of lower-sideband up-converters, and

$$f = f_0' - f^p \tag{4}$$

for the case of upper-sideband up-converters. In both cases, the amplifiers will yield gain. The lower-sideband type introduces some negative-resistance amplification in addition to the up-converter amplification,[2,3] however, and will therefore generally give more gain. For gain to be achieved, the variable-capacitance diode must see proper terminations at both frequencies, f and f_0'. Since f_0' is a fixed frequency, it should be relatively easy to maintain proper termination at that frequency. The input frequency f varies, however, and the tuning range of the amplifier will be determined largely by the range of f for which proper terminating conditions can be maintained.

B. Factors Permitting Large Tuning Range

For convenience, let us first consider the case of an upper-sideband up-converter with a fixed pump frequency. We analyze a wide-band, upper-sideband up-converter using a fixed pump frequency f^p, with an input-impedance-matching filter whose pass band is centered at f_0 and an output-impedance-matching filter whose pass band is centered at f_0'. In order to obtain maximum operating bandwidth, it is necessary that the condition

$$\frac{w}{w'} = \frac{f_0'}{f_0} \tag{5}$$

be satisfied, where w is the fractional bandwidth $\Delta f/f_0$ of the input filter and w' is the fractional bandwidth $\Delta f'/f_0'$ of the output filter.[1] Eq. (5) says, in effect, that the input and output bands Δf and $\Delta f'$, respectively, must be equal. From the theory of wide-band up-converters,[1] it is seen that if, for example, Δf were made larger than $\Delta f'$, the output filter bandwidth $\Delta f'$ would have to be shrunk to be *smaller* than it could be if Δf and $\Delta f'$ were made to be equal. Since the smaller bandwidth will be the one that limits the over-all transmission, the operating bandwidth necessarily becomes smaller if Δf and $\Delta f'$ are made unequal. Thus it is seen that for the usual case where the pump frequency is fixed, wide-band impedance matching must be accomplished both at the signal input and also at the upper-sideband output band, and best results are obtained if the bandwidth for impedance match is equal for both of these bands.

For the tunable up-converters considered herein, a quite different situation exists. In this case, narrow rather than wide bandwidth is desired. A wide-band impedance match is desired at the input channel, however, so that the input acceptance band can be moved about by varying the pump frequency. Under these conditions, we may expand the input bandwidth Δf and be quite happy with the required shrinkage of the sideband output bandwidth $\Delta f'$. With such an amplifier, the bandwidth $\Delta f'$ of the upper-sideband filter will be the operating bandwidth of the amplifier, while Δf will become the tuning range. If there were no practical considerations, such as the effects of losses and the need for practical impedance levels, the tuning range Δf could be made to approach infinity while the output bandwidth $\Delta f'$ would approach zero; practical considerations, however, do limit the obtainable tuning range. Nevertheless, it is clear that the *tuning range* obtainable with such a device is considerably greater than the *bandwidth* obtainable in an up-converter using a fixed pump frequency. As is seen in Matthaei,[1] bandwidths of the order of 16 per cent appear to be feasible for upper-sideband up-converters having fixed pump frequencies. The tuning range which is feasible for up-converters with varying pump frequencies is thus likely to be several times this value.

An additional problem, not yet mentioned, arises in this case of an up-converter with varying pump frequency. Additional impedance matching for the pump input channel must be provided in order to pump the diode satisfactorily at frequencies throughout the required pump frequency range. It can be shown that it will generally be impossible to obtain a good match between the pump signal oscillator and the small resistance in the diode while tuning across the required pump frequency band. However, it is possible to design impedance-matching filters which will give a nearly uniform, minimum reflection level throughout the required band. Then it is merely necessary to have a pump oscillator with sufficient available power to compensate for the reflection loss.

The discussion above has been phrased in terms of upper-sideband up-converters. The same principles,

[2] J. M. Manley and H. E. Rowe, "Some general properties of nonlinear elements—part 1. General energy relations," Proc. IRE, vol. 44, pp. 904–913; July, 1956.

[3] H. E. Rowe, "Some general properties of nonlinear elements. II. Small signal theory," Proc. IRE, vol. 46, pp. 850–860; May, 1958.

however, apply to lower-sideband up-converters. The main difference between the two devices is that the lower-sideband type tends to reflect a negative resistance through the coupling action of the varying capacitance. This negative resistance has the advantage that it contributes to the gain of the amplifier, but it also has the disadvantage that it could lead to oscillation if the amplifier were improperly terminated. Calculations indicate that good results should be obtainable by keeping the negative resistance small, so that stability will not be a problem.

II. Definition of Diode Parameters

The specification of the diode parameters will be the same as that used previously,[1,3] but will be summarized here for easy reference. The diode capacitance parameters C_0 and C_1 are defined by

$$C(t) = C_0 + 2C_1 \cos(2\pi f^p t + \phi_1) + \cdots, \tag{6}$$

where $C(t)$ is the time variation of the pumped diode. If the diode is to be resonated in shunt, it is convenient to use the short-circuit admittances[1,3]

$$B_{11} = 2\pi f C_0, \qquad B_{12} = 2\pi f C_1$$
$$B_{21} = 2\pi f' C_1, \qquad B_{22} = 2\pi f' C_0. \tag{7}$$

In the discussion to follow, subscript zeros, such as

$$(B_{11})_0, \qquad (B_{12})_0, \qquad (B_{21})_0, \qquad (B_{22})_0,$$

will be used to indicate parameters evaluated at the signal input midband frequency f_0, or at the sideband output midband frequency f_0', whichever is appropriate for the given parameter.

The equivalent shunt diode loss conductances are defined as

$$G_d = \frac{2\pi f_0 C_0}{Q_d} = \frac{(B_{11})_0}{Q_d} \tag{8}$$

seen by input signal components and

$$G_d' = \frac{2\pi f_0' C_0}{Q_d'} = \frac{(B_{22})_0}{Q_d'} = G_d\left(\frac{f_0'}{f_0}\right)^2 \tag{9}$$

seen by the sideband components. Q_d is the operating Q of the diode at frequency f_0, and Q_d' is the operating Q at f_0'. Since the diode loss resistance is effectively in series with the capacitance,

$$Q_d' = Q_d \frac{f_0}{f_0'} \cdot \tag{10}$$

Analogously, if the diode is to be resonated in series, it is convenient to use the open-circuit impedances[1]

$$X_{11} = \frac{1}{2\pi f C_0{}^s}, \qquad X_{12} = \frac{1}{2\pi f' C_1{}^s}$$
$$X_{21} = \frac{1}{2\pi f C_1{}^s}, \qquad X_{22} = \frac{1}{2\pi f' C_0{}^s} \tag{11}$$

where

$$C_0{}^s = C_0(1 - a^2), \qquad C_1{}^s = \frac{C_0(1 - a^2)}{a} = \frac{C_0{}^s}{a} \tag{12}$$

and

$$a = \frac{C_1}{C_0} \cdot \tag{13}$$

In this case, the effects of losses are represented by series resistors

$$R_s = R_s' = \frac{1}{Q_d 2\pi f_0 C_0{}^s} = \frac{(X_{11})_0}{Q_d} = \frac{(X_{22})_0}{Q_d'}, \tag{14}$$

which are the same at both the signal and sideband frequencies.

III. Definition of Filter Parameters

In order to reduce the number of degrees of freedom involved in the amplifier design, the design of the various band-pass filters required will be based on the use of lumped-element, low-pass prototypes. When these low-pass prototypes have been specified, the only parameters which remain to be specified in the corresponding band-pass filters are their center frequencies, their impedance levels, and their fractional bandwidths. Their center frequencies, of course, will be determined by the desired operating bands. The impedance level of the input and pump channel filters will be determined from considerations of their desired bandwidths and the impedance-matching properties which are required.

Fig. 1 shows a typical low-pass prototype and a typical Tchebycheff response for such a filter. Tables of normalized element values for Tchebycheff and maximally flat low-pass filters of this kind are presently available[4,5] Fig. 2 shows the corresponding band-pass filter obtained by the standard low-pass to band-pass transformation given in Fig. 2(a).

In an actual microwave device it will usually be necessary to use resonators which are a combination of semi-lumped and transmission-line elements rather than lumped-element resonators such as are shown in Fig. 2. For this reason, it is convenient to use the equivalent representation shown in Fig. 3. In this case, the actual nature of the resonators is left unspecified, but it is assumed that in the vicinity of f_0 they exhibit resonance characteristics similar to those of the resonators in Fig. 2 [as is implied by Fig. 3(a) and (d)]. Then the properties of the individual resonators are specified in terms of the resonant frequency f_0 and a *slope parame-*

[4] L. Weinberg, "Additional tables for design of optimum ladder networks, Pts. 1 and 2," *J. Franklin Inst.*, vol. 264, pp. 7–23, 127–138; July, August, 1957.

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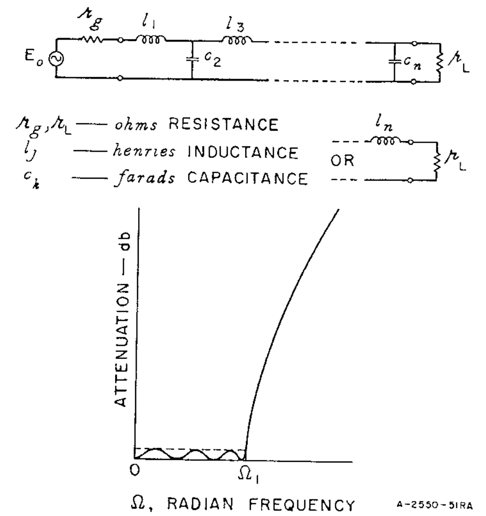

Fig. 1—Low-pass prototype filter and a typical Tchebycheff response.

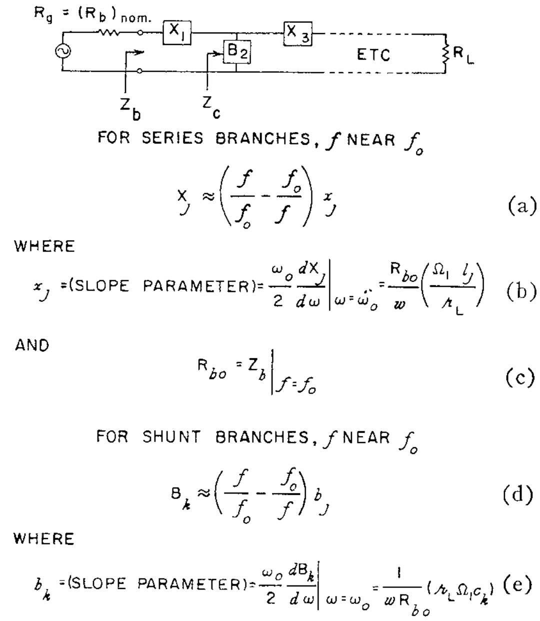

Fig. 3—General description of band-pass filters in terms of resonator slope parameters (Ω_1, l_j, c_k and r_L are defined in Fig. 1).

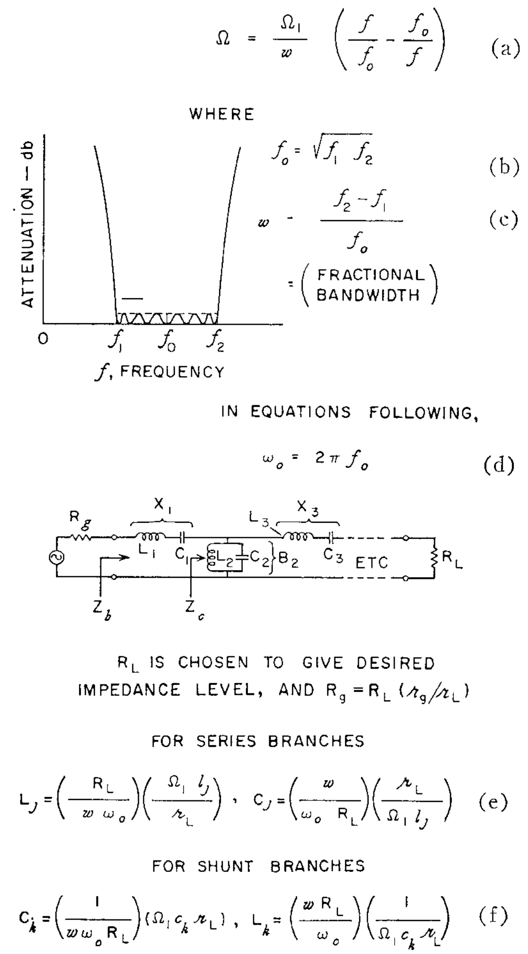

Fig. 2—Summary of relations for design of lumped-element band-pass filters from low-pass prototypes.

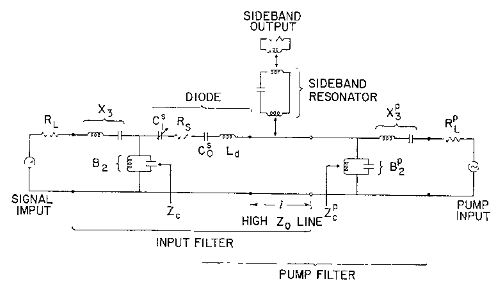

Fig. 4—A possible circuit for an up-converter for electronically-tunable filter applications.

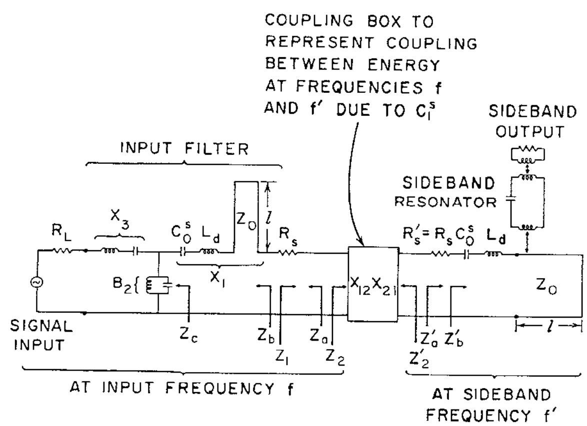

Fig. 5—A circuit which is approximately equivalent to that in Fig. 4 for energy components at the input frequency f or the sideband frequency f'.

ter. For a series, lumped-element resonator, the *reactance slope parameter* is simply

$$X_j = \omega_0 L_j = \frac{1}{\omega_0 C_j} = \frac{\omega_0}{2} \left.\frac{dX_j}{d\omega}\right|_{\omega_0}, \tag{15}$$

where $\omega_0 = 2\pi f_0$ and X_j is the reactance of the resonator circuit. The derivative form in (15) and in Fig. 3(b) gives the corresponding reactance slope parameter for *any* series-resonant circuit, whether of lumped-element form or not. As indicated in Fig. 3(e), for circuits exhibiting a shunt resonance an analogous *susceptance slope parameter* b_k applies. Fig. 3(b) and (e) relates the band-pass filter resonator slope parameters to the elements of the low-pass prototype.

The parameter R_{b0} defined in Fig. 3(c) equals R_L for the circuit as shown. However, the definition given for R_{b0} is introduced because some microwave filters give impedance-level transformations so that R_{b0} may not necessarily equal R_L.

IV. An Up-Converter Model for Purposes of Analysis and Discussion

The up-converter model discussed in this section incorporates some features which have been used at SRI quite successfully on a degenerate parametric amplifier, and it is believed that they could also be used to advantage in electronically tunable up-converters in certain frequency ranges. However, the design equations and charts in the following sections should be applicable for any of a variety of coaxial, stripline, or waveguide types of up-converter structures. In this model, the diode will be resonated in series, although the design data to follow can be applied also for designs where the diode is resonated in shunt.

Fig. 4 shows the model up-converter circuit. This circuit uses a three-resonator wide-band input filter, a three-resonator moderately wide-band pump circuit filter, and a narrow-band, single-resonator output filter. The diode plus the length of high-impedance transmission line introduces multiple resonances, one of which provides the resonance for the No. 1 resonator of the input filter, and another of which provides the resonance for the No. 1 resonator of the pump filter. Note that the diode representation includes C_0^s, C_1^s, and R_s in series along with L_d, the diode internal inductance. Since the number of resonators for the signal-input and pump filters may vary under different circumstances, it is convenient for both the input and pump filters to define the resonator formed by the diode circuit as the No. 1 resonator, even though the generator is actually at the other end. Under these conditions, R_L becomes the generator internal impedance instead of R_g, but the power transmission properties are unaffected since the filter circuit obeys the reciprocity theorem.

The performance of the circuit in Fig. 4 can best be understood by considering its operation at the various frequencies of interest. Fig. 5 shows the equivalent circuit of the input filter circuit at frequency f and the output circuit at the sideband frequency f', with a box labeled $X_{12}X_{21}$ to represent the coupling effect of C_1^s between energy components at frequencies f and f'. The operation of this box is different for lower-sideband up-converters than for upper-sideband up-converters,[1] and its operation will be summarized in Sections V and VII. Since the pump filter center frequency f_0^p is much higher than the center frequency f_0 of the input filter, the susceptance B_2^p of the No. 2 resonator of the pump filter in Fig. 4 will be very large, so that the high-impedance line is effectively short-circuited. At frequency f_0, the high-impedance line is less than $\lambda/8$ long and acts much like a lumped inductance which brings the diode to series resonance at that frequency. Since C_0^s, L_d, R_s, the impedance of the short-circuited line, and the impedance reflected through C_1^s (Z_2 in Fig. 5) are effectively in series, the diode and high Z_0 line together may be represented as shown in Fig. 5 by R_s, Z_2, and the series resonator X_1.

At the pump-channel band-center frequency f_0^p, the impedance Z_c in Fig. 4 will be reactive and nearly zero, and the diode plus the high Z_0 line will again exhibit resonance. The impedance seen looking into the high Z_0 line from the right in Fig. 4 is given by

$$Z = \frac{(R_s + jX_d) + jZ_0 \tan\theta}{1 + \dfrac{j(R_s + jX_d)\tan\theta}{Z_0}}, \tag{16a}$$

where $R_s + jX_d$ is the total impedance developed across the diode along with the input filter reactance Z_c in Fig. 4 at the pump frequency f^p. Choosing Z_0 to be large as compared to R_s (a condition which is easy to fulfill since R_s might typically be around 4 ohms), to a good approximation (16a) becomes

$$Z = R_g^p + jX_1^p \approx \frac{R_s}{\left(1 - \dfrac{X_d}{Z_0}\tan\theta\right)} + \frac{j(Z_0 \tan\theta + X_d)}{\left(1 - \dfrac{X_d}{Z_0}\tan\theta\right)}. \tag{16b}$$

In most cases, the pump band center f_0^p will be above the self-resonant frequency of the diode; therefore, at f_0^p the reactance X_d will usually be inductive, and θ will be somewhat less than π. Under these conditions, the quantity in the denominators of (16b) will be somewhat greater than one (since $\tan\theta$ is negative), and will be slowly varying since X_d/Z_0 will be increasing with frequency, while $\tan\theta$ will be decreasing in magnitude as the frequency increases.

Since θ will be roughtly of the order of π, in some cases the denominators of the terms in (16b) will be very nearly one. Assuming for the moment that this is the case and that Z_c for the input filter in Fig. 4 is negligible at f_0^p, then (16b) is equivalent to the impedance of R_s, C_0^s, L_d, and the short-circuited, high Z_0 line all connected in series as shown in Fig. 6(a). If the denominator in (16b) cannot be neglected, the magnitude of $R_g^p+jX_1^p$ will be affected as indicated by the equation, but the general nature of the performance will be qualitatively the same.

It is thus seen that for frequencies in the vicinity of f_0 and f_0^p, C_0^s, L_d and the high Z_0 line operate at least qualitatively in a manner similar to the series-resonant circuit shown in Fig. 6(a), which has the reactance characteristics shown in Fig. 6(b). The reactance slope in the vicinity of f_0 will be important in the design of the signal-input filter, while the reactance slope in the vicinity of f_0^p will be important in the design of the pump input filter. At the sideband frequency f_0', the diode and high Z_0 line in Fig. 4 will see reactive impedances Z_c and Z_c^p of small magnitude, but the diode and line will not be resonant. However, the presence of the lightly-coupled sideband resonator in Fig. 4 will introduce a narrow resonance at either the lower- or upper-sideband frequencies indicated in Fig. 6(b). It is desirable that this sideband-output resonator be coupled as lightly as possible, consistent with low transmission loss, so that it will cause a minimum increase in the reactance slope in the vicinity of f_0^p. Since the up-converter would probably be followed by a fixed-tuned, superheterodyne receiver, an additional high-Q filter following the sideband-output filter might be desirable to ensure high attenuation at the image frequency of the superheterodyne receiver.

Fig. 7 shows portions of a possible strip transmission line amplifier of the type under consideration. The diode used is in a computer diode package having a glass envelope and wire leads. (The Hughes 1N896 diodes are examples of varactor diodes packaged in this manner.) The wire diode leads then serve as the high Z_0 line for resonating the diode. Resonators Nos. 2 and 3 for the signal-input filter are realized as two semi-lumped shunt resonators separated by a quarter-wavelength (at frequency f_0) of line. The quarter-wavelength line eliminates the need to construct resonator No. 3 as a series resonator. The signal-input filter would be of wide bandwidth and it could be designed using the theory in Sections V or VII along with techniques for design of wide-band filters which have been previously treated.[6]

The pump frequency must be able to shift an amount equal to the bandwidth of the signal-input filter, but since the pump frequency band is centered considerably higher than that of the signal-input band, the pump filter fractional bandwidth will be relatively small. Pump filter resonators Nos. 2 and 3 are realized as quarter-wavelength direct-coupled, two-port resonators, since this construction is convenient for narrow or moderate bandwidths, and will not have any spurious pass bands until about three times the frequency of the first pass band.[7] Each resonator bar is series-capacitance-

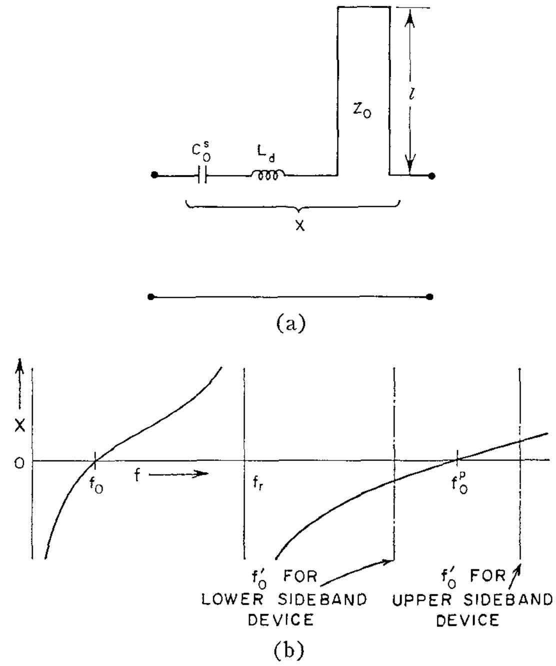

Fig. 6—(a) Approximate equivalent circuit for the diode resonator. (b) Reactance properties of the resonator in Fig. 4.

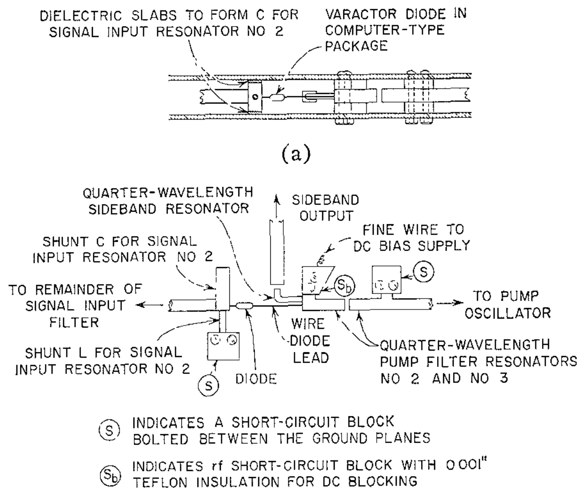

Fig. 7—A possible strip-transmission-line embodiment of the circuit in Fig. 4. The input, pump, and sideband filters are realized in practical microwave filter structures which will appear from the diode resonator circuit to be equivalent to those in Fig. 4 for the frequencies of interest. (a) Cutaway side view. (b) Top view with cover plate removed.

[6] G. L. Matthaei, "Design of wide-band (and narrow-band) band-pass microwave filters on the insertion loss basis," IRE Trans. on Microwave Theory and Techniques, vol. MIT-8, pp. 580–593; November, 1960.

[7] G. L. Matthaei, "Direct-coupled, band-pass filters with $\lambda_0/4$ resonators," 1958 IRE National Convention Record, pt. 1, pp. 98–111.

coupled at one end and shunt-inductance-coupled at the other. Looking to the right from the high Z_0 wire in the diode circuit, the impedance of the structure appears much like Z_c^p seen looking to the right from the high Z_0 line in Fig. 4 for frequencies in the vicinity of f_0^p.

The sideband-output resonator shown in Fig. 7 is one-quarter-wavelength long; it is inductively coupled to the diode and high Z_0 wire circuit, but series-capacitance-coupled to the sideband-output line.

The circuit shown in Fig. 7 appears to be a possible realization of an electronically-tunable up-converter having input frequencies centered around 1000 Mc. However, other configurations are also possible.

V. Determination of Signal-Input and Sideband-Output Circuit Parameters for Upper-Sideband Up-Converters

When power is taken out at the upper sideband while the other components of the mixing process are suppressed, then the $X_{12}X_{21}$ coupling box in Fig. 5 operates like an impedance inverter so that[1]

$$Z_2 = \frac{X_{12}X_{21}}{Z_a'} \tag{17}$$

and

$$Z_2' = \frac{X_{12}X_{21}}{Z_a}, \tag{18}$$

where, as indicated in Fig. 5, the unprimed impedances are evaluated at the signal-input frequency f, while the primed impedances are evaluated at the sideband frequency f'. By the Manley-Rowe equations,[2] the net power P_2 *entering* the left side of $X_{12}X_{21}$ is related to the net power P_2' *entering* the right side of $X_{12}X_{21}$ by

$$\frac{P_2'}{P_2} = -\frac{f'}{f}, \tag{19}$$

where the minus sign indicates that if power flows into the left side, power will flow out of the right side.

Let us define $(R_b)_{\text{nom}}$ as the nominal value of the signal-input filter impedance Z_b in Fig. 5. The impedance $(R_b)_{\text{nom}}$ is purely real and is equal to the resistive terminating impedance which would give best transmission through the fiter. In terms of Fig.3,

$$(R_b)_{\text{nom}} = R_g = \frac{r_g}{r_L} R_{b0} \tag{20}$$

where r_g and r_L are from the lumped-element prototype. For purposes of computing the nominal gain of the amplifier, the signal-input filter will be replaced by a Thevenin equivalent generator having an internal impedance equal to $(R_b)_{\text{nom}}$. At the sideband-output frequency f_0', the impedance Z_b' in Fig. 5 will be purely real and will be defined as

$$R_{b0}' = Z_b'\big|_{f'=f_0'}. \tag{21}$$

Using (17)–(19) and the above definitions for $(R_b)_{\text{nom}}$ and R_{b0}', it can easily be shown that the nominal power gain for input signals in the vicinity of f_0 is

$$\frac{P'_{\text{out}}}{P_{\text{avail}}} = \frac{f_0'}{f_0}\,\frac{4(R_b)_{\text{nom}}R_{b0}'(X_{12}X_{21})_0}{\{[(R_b)_{\text{nom}} + R_s](R_{b0}' + R_s') + (X_{12}X_{21})_0\}^2}, \tag{22}$$

where $R_s = R_s'$ is the diode loss resistance, P_{avail} is the available power at the input frequency f_0, and P_{out}' is the nominal output power at the sideband frequency f_0'. The gain will vary somewhat across the tuning range due to variations in Z_b, $X_{12}X_{21}$, and f'/f, but (22) gives what will herein be referred to as the *nominal gain* of an upper-sideband up-converter.

In the design of an amplifier, the input and pump channel filters serve primarily as wide-band impedance-matching networks. Thus, the first step in the design process is to design the diode resonator circuit and then determine the slope parameters x_1 and x_1^p at frequencies f_0 and f_0^p, respectively, by use of the derivative form in Fig. 3(b). By Fig. 3(b), for a desired fractional bandwidth w of the input circuit, R_{b0} must be

$$R_{b0} = x_1 w\left(\frac{r_L}{\Omega_1 l_1}\right), \tag{23}$$

and then $(R_b)_{\text{nom}}$ is given by (20). The slope parameters x_j and b_k for the other resonators of the input filter may then be computed from the lumped-element prototype filter parameters, R_{b0}, and w, by use of Fig. 3(b) and (e).

Knowing $(R_b)_{\text{nom}}$ and f_0'/f_0, and having values for $(X_{11})_0$, $R_s = R_s'$, and $a = C_1/C_0$ for the diode, the maximum gain will be obtained if R_{b0}' matches Z_2' in Fig. 5, which requires that

$$R_{b0}' = R_s' + \frac{(X_{12}X_{21})_0}{(R_b)_{\text{nom}} + R_s} \tag{24}$$

be satisfied. This can be expressed as

$$R_{b0}' = R_s'\left[1 + \frac{1}{v^2(T+1)}\right], \tag{25}$$

where

$$T = \frac{(R_b)_{\text{nom}}}{R_s} = \frac{(R_b)_{\text{nom}}Q_d}{(X_{11})_0} \tag{26}$$

and

$$v = \frac{1}{aQ_d}\sqrt{\frac{f_0'}{f_0}}. \tag{27}$$

Inserting (25) in (22) yields, after some manipulation,

$$\frac{\dfrac{P'_{\text{out}}}{P_{\text{avail}}}}{\dfrac{f_0'}{f_0}} = \frac{T}{(T+1)[v^2(T+1)+1]}. \tag{28}$$

If this analysis is carried out on the dual basis for the case of the diode resonated in shunt, we find the dual expressions

$$G_{b0} = b_1 w\left(\frac{r_L}{\Omega_1 l_1}\right), \tag{29}$$

$$(G_b)_{\text{nom}} = G_g = \frac{r_g}{r_L} G_{b0}, \tag{30}$$

$$G_{b0}' = G_d'\left[1 + \frac{1}{v^2(T+1)}\right], \tag{31}$$

$$T = \frac{(G_b)_{\text{nom}}}{G_d} = \frac{(G_b)_{\text{nom}} Q_d}{(B_{11})_0}, \tag{32}$$

and (27) and (28) apply as before. The parameter G_d' in (31) is again the equivalent shunt diode loss conductance given by (9). In order to facilitate design calculations, a plot of $(P_{\text{out}}'/P_{\text{avail}})/(f_0'/f_0)$ vs v is shown in Fig. 8 for various values of T. The reference form for the band-pass filters is the dual of that in Fig. 3; the slope parameters and terminations for this dual filter may be conveniently specified directly in terms of the low-pass prototype in Fig. 1.

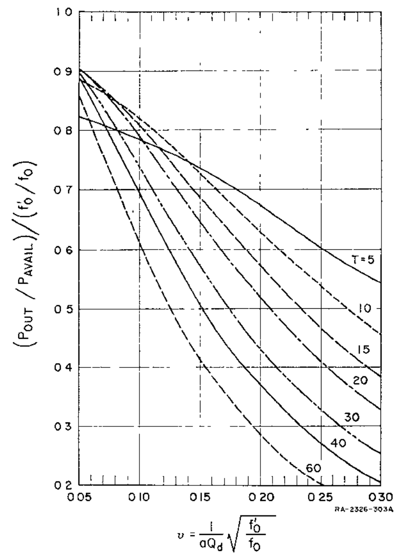

Fig. 8—Chart for determining the gain of upper-sideband up-converters. [$T = (R_b)_{\text{nom}}/R_s$ for the case of a diode resonated in series, and $T = (G_b)_{\text{nom}}/G_d$ for the case of a diode resonated in shunt. This chart does not apply accurately for the shunt resonance case if the diode has significant series inductive reactance.]

VI. Pump Circuit Impedance-Matching Filter

In order that the pump channel have a nearly uniform reflection loss across the pump frequency band, it is necessary to design an impedance-matching filter as shown in Fig. 9. In this case, the resistance R_g^p is given by (16b). The No. 1 resonator, having a reactance X_1^p [given by (16b) for the case of Figs. 4, 7 and 9] is characterized by the slope parameter x_1^p computed at frequency f_0^p in Fig. 6(b) using the derivative form in Fig. 3(b). (Recall that in this case, the resonance of the No. 1 resonator is created by the second series resonance of the diode circuit, as discussed in Section IV.) The fractional bandwidth w_p of the pump filter is

$$w_p = w\frac{f_0}{f_0^p}, \tag{33}$$

where w is again the desired fractional bandwidth of the input circuit. By Fig. 3(b),

$$\left(\frac{\Omega_1 l_1}{r_L}\right)_p = \frac{x_1^p w_p}{R_{b0}^p}, \tag{34}$$

where the subscript p on $(\Omega_1 l_1/r_L)_p$ is introduced to indicate that these parameters refer to the low-pass prototype for design of the pump filter. Since, in accord with Figs. 1 and 3,

$$\frac{R_{b0}^p}{R_g^p} = \left(\frac{r_L}{r_g}\right)_p, \tag{35}$$

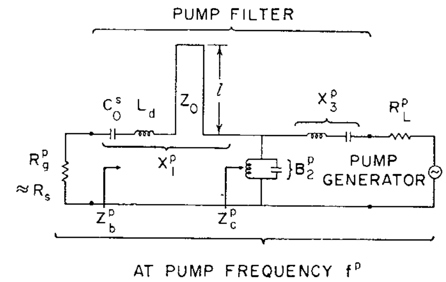

Fig. 9—A circuit approximately equivalent to that in Fig. 4 for energy at the pump frequency f^p.

we may write (34) as

$$\left(\frac{\Omega_1 l_1}{r_g}\right)_p = \frac{x_1^p w_p}{R_g^p}, \tag{36}$$

where, for the case of Figs. 4, 7 and 9, R_g^p is given by (16b). The problem now focuses on finding a low-pass prototype having $(\Omega_1 l_1/r_g)_p$ as given by (36), with a desired amplitude of Tchebycheff pass-band attenuation ripple, and with a minimum amount of reflection loss in the pass band. We are stuck with the prototype impedance $j\Omega l_1 + r_g$, and the other elements of the prototype filter are introduced to optimize the impedance across the band $\Omega = 0$ to $\Omega = \Omega_1$.

Certain aspects of impedance-matching problems of this type were treated by Bode.[8] Fano treated the general limitations on lossless impedance matching, and also the design of certain Tchebycheff impedance-

[8] H. W. Bode, "Network Analysis and Feedback Amplifier Design," D. Van Nostrand Co., Inc., New York, N. Y., pp. 363–368; 1945.

matching networks,[9] and Carlin,[10] Green,[11] Barton,[12] Matthaei,[13] and others have made various contributions.

The problem at hand can be treated as follows. Suppose that it is desired to use n resonators in the pump-circuit filter and that a Tchebycheff ripple of T_r db is specified. Compute

$$H = \text{antilog}_{10} \frac{T_r}{10}, \tag{37}$$

$$d = \sinh\left[\frac{\sinh^{-1}\sqrt{\dfrac{1}{H-1}}}{n}\right], \tag{38}$$

and

$$e = d - 2\left(\frac{r_g}{\Omega_1 l_1}\right)\sin\frac{\pi}{2n}. \tag{39}$$

Then, for given $r_g/(\Omega_1 l_1)$, T_r, and n, the reflection coefficient value at the maximum reflection points of the Tchebycheff pass band is given by

$$|\rho|_{\max} = \frac{U_n(e)}{U_n(d)}, \tag{40}$$

where $U_n(e)$ and $U_n(d)$ are polynomials tabulated by Matthaei[13] and plotted in Fig. 10 of this paper. Under these conditions, the minimum pump power delivered to the diode in the pump filter pass band is

$$A = \frac{P_{\text{transmitted}}}{(P_{\text{avail}})^p} = 1 - |\rho|^2_{\max} \tag{41}$$

times the available power $(P_{\text{avail}})^p$ of the pump signal generator. It will be found that the larger the slope parameter x_1^p and the larger the fractional bandwidth w_p, the smaller A must be and the more available power will be required.

Having values for $r_g/(\Omega_1 l_1)$, T_r, and $|\rho|_{\max}$ for the pump filter, the actual element values of the low-pass prototype are in this case obtained most easily by use of equations given by Green.[11,14] It should be understood that (37)–(41) apply only to simple ladder circuits such as that in Fig. 1 or their band-pass equivalents (such as that in Fig. 9).

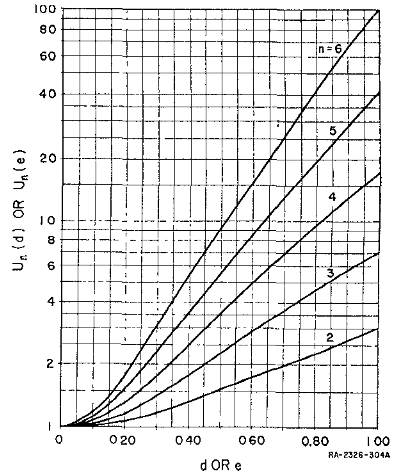

Fig. 10—Chart for use in determining the reflection loss required in the pump circuit when using an optimum Tchebycheff impedance-matching filter.

VII. Determination of Signal-Input and Sideband-Output Circuit Parameters for Lower-Sideband Up-Converters

If power is taken out at the lower sideband instead of at the upper sideband, (17)–(19) become, respectively,

$$Z_2 = -\frac{X_{12}X_{21}}{(Z_a')^*}, \tag{42}$$

$$Z_2' = -\frac{X_{12}X_{21}}{(Z_a)^*}, \tag{43}$$

and

$$\frac{P_2'}{P_2} = \frac{f'}{f}, \tag{44}$$

where the asterisk indicates the complex conjugate. Because of the negative sign in (42), this type of operation gives gain at the input frequency f, due to negative input resistance. As is implied by (44), the power P_2' (at the lower-sideband frequency f') going out the right side of $X_{12}X_{21}$ in Fig. 5 is larger than the power P_2 at frequency f going out the left side by the factor f'/f. Thus, whatever gain is achieved at the input frequency is increased at the $X_{12}X_{21}$ box by the factor f'/f for power taken out at the sideband frequency f'.

In this case, (20) and (21) apply as before, but (22) becomes

$$\frac{P'_{\text{out}}}{P_{\text{avail}}} = \left(\frac{f_0'}{f_0}\right)\frac{4(R_b)_{\text{nom}}R_{b0}'(X_{12}X_{21})_0}{\left\{[(R_b)_{\text{nom}} + R_s](R_{b0}' + R_s') - (X_{12}X_{21})_0\right\}^2}. \tag{45}$$

[9] R. M. Fano, "Theoretical limitations on the broadband matching of arbitrary impedances," *J. Franklin Inst.*, vol. 249, pp. 57–83, 139–154; January and February, 1950.

[10] H. J. Carlin, "Synthesis techniques for gain-bandwidth optimization in passive transducers," Proc. IRE, vol. 48, pp. 1705–1714; October, 1960.

[11] E. Green, "Amplitude-Frequency Characteristics of Ladder Networks," Marconi's Wireless Telegraph Co., Ltd., Chelmsford, Essex, England, pp. 62–78; 1954.

[12] B. F. Barton, "Design of Efficient Coupling Networks," Electronic Defense Group, University of Michigan, Ann Arbor, Tech. Rept. No. 44, Contract DA 36-039 SC-63203; March, 1955.

[13] G. L. Matthaei, "Synthesis of Tchebycheff impedance-matching networks, filters, and interstages," IRE Trans. on Circuit Theory, vol. CT-3, pp. 162–172; September, 1956.

[14] E. Green, "Synthesis of ladder networks to give Butterworth or Chebyshev response in the pass band," *Proc. IEE*, pt. 4, Monograph no. 88; 1954.

which gives the nominal gain of the amplifier. The value of R_{b0} for the input filter is determined by (23) as before, but R_{b0}' cannot be determined by impedance matching to maximize the gain as was done to obtain (24), since the maximum possible gain is now infinite (yielding oscillation). In this case, we define a stability parameter

$$D = \frac{(R_b)_{\text{nom}} + R_s}{|Z_2|_{f=f_0}} = \frac{(R_b)_{\text{nom}} + R_s}{\left[\dfrac{(X_{12}X_{21})_0}{R_{b0}' + R_s'}\right]}, \qquad (46)$$

which is seen to be the ratio of the nominal positive resistance of Z_a in Fig. 5 to the magnitude of the negative resistance Z_2 seen looking in the opposite direction. Parameter D fixes the nominal value of the negative resistance gain of the amplifier, and for stability, D must be greater than one. By (46)

$$R_{b0}' = \frac{D(X_{12}X_{21})_0}{(R_{b0} + R_s)} - R_s' \qquad (47)$$

$$= R_s'\left[\frac{D}{v^2(T+1)} - 1\right], \qquad (48)$$

where T and v have the same meaning as in (26) and (27), respectively. Substituting (48) in (45) gives

$$\frac{\dfrac{P'_{\text{out}}}{P_{\text{avail}}}}{\dfrac{f_0'}{f_0}} = \frac{4\left[D - \left(v^2T + \dfrac{D}{T+1}\right)\right]}{(D-1)^2}. \qquad (49)$$

The ratio $(P_{\text{out}}'/P_{\text{avail}})/(f_0'/f_0)$ is plotted vs v in Fig. 11(a)-(c) for $D=2$, 3, 4 and various T values. These charts may also be used for the dual case where the diode is resonated in shunt by defining T as in (32). For the dual shunt-diode case, (48) becomes the dual equation

$$G_{b0}' = G_d'\left[\frac{D}{v^2(T+1)} - 1\right]. \qquad (50)$$

VIII. Estimated Performance of Some Design Examples

Let us estimate the performance of some up-converters having an input band center of $f_0 = 1$ kMc. The construction in Figs. 4 and 7 will be assumed using a Hughes 1N896 diode which has the computer-type package shown in Fig. 7. The lead wires on the diode will serve as the high Z_0 line. The wires are 0.020 inch in diameter, and if a 0.500-inch ground-plane spacing is used, they will have an impedance of 207 ohms. The diode will be assumed to have a C_0 value of 1.2 $\mu\mu$f (corresponding to a zero bias capacitance of about 2.3 $\mu\mu$f). The series inductance L_d of the diode itself will be taken as 4×10^{-9} h, and the operating Q will be taken as

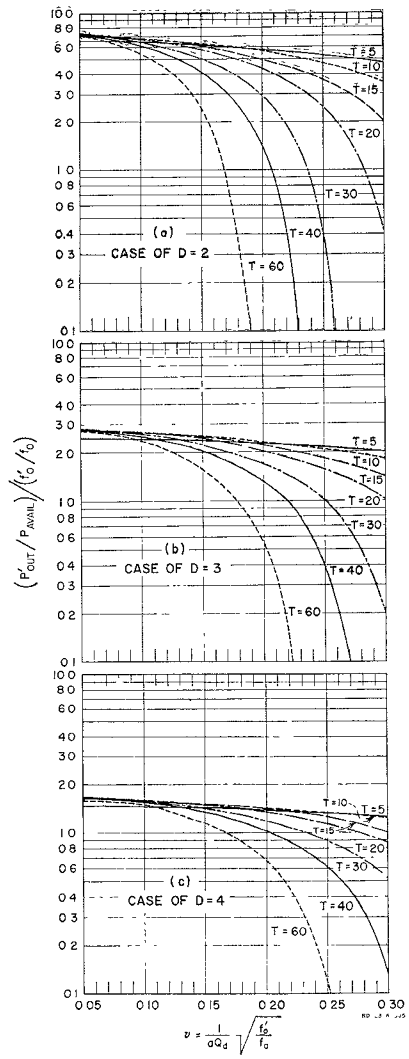

Fig. 11—Charts for determining the gain of lower-sideband up-converters. [$T=(R_b)_{\text{nom}}/R_s$ for the case of a diode resonated in series, and $T=(G_b)_{\text{nom}}/G_d$ for the case of a diode resonated in shunt. Parameter D is the ratio of the magnitude of positive resistance to negative resistance seen at the diode (or for the shunt case, positive conductance to negative conductance). These charts do not apply accurately for the shunt resonance case if the diode has significant series inductive reactance.]

$Q_d = 35$ at $f_0 = 1$ kMc (from the manufacturer's data, this appears to correspond to a cutoff frequency of around 70 kMc for this type of abrupt junction diode). The $a = C_1/C_0$ parameter of the diode will be taken to be 0.25.

In terms of the approximate equivalent circuit in Fig. 6(a), for the diode resonator,

$$X(f) = \frac{-1}{2\pi f C_0^s} + 2\pi f L_d + Z_0 \tan\left(\frac{\pi f}{2f_r}\right), \qquad (51)$$

where f_r is the frequency shown in Fig. 6(b) for which the high Z_0 line is $\lambda/4$ long. Setting $f=f_0=10^9$, $C_0^s = C_0(1-a^2) = 1.17\times10^{-12}$fd, $L_d = 4\times10^{-9}$h, and $Z_0 = 207$, we find that $f_r = 3.21$ kMc is required, which in turn calls for the $Z_0 = 207$-ohm line to be $l = 0.922$ inch.

long. The slope parameter x_1 at frequency f_0 is obtained from

$$x_1 = \frac{\omega_0}{2} \frac{dX}{d\omega}\bigg|_{\omega=2\pi f_0=\omega_0}$$

$$= \frac{1}{2}\left[\frac{1}{2\pi f_0 C_0^s} + 2\pi f_0 L_d + \left(\frac{\pi f_0}{2f_r}\right)\frac{Z_0}{\cos^2\left(\frac{\pi f_0}{2f_r}\right)}\right], \quad (52)$$

which gives $x_1 = 145$ ohms. Solution of (51) shows that f_0^p in Fig. 6(b) will occur at about $f_0^p = 5.45$ kMc, and by calculations similar to (52), the slope parameter there is about $x_1^p = 430$ ohms.[15]

Let us first consider an upper-sideband up-converter having an input tuning band-edge ratio of one-half octave (*i.e.*, for the input filter, $f_2/f_1 = \sqrt{2}$ where f_1 and f_2 are defined in Fig. 2). This calls for $w = 0.348$. The input filter will be assumed to have $n = 3$ resonators and 1-db Tchebycheff ripple. From tables [4,5] of low-pass prototype filter elements, we find that $(\Omega_1 l_1/r_g) = 2.024$ and $r_g = r_L$ for this case. Then by (20) and (23), we obtain $(R_b)_{\text{nom}} = R_{b0} = 25$ ohms. Since $R_s = (X_{11})_0/Q_b = 3.86$ ohms, $T = (R_b)_{\text{nom}}/R_s = 6.5$. In this upper-sideband case, $f_0' = f_0^p + f_0 = 5.45 + 1 = 6.45$ kMc which gives, by (27), $v = 0.291$. By use of T and v in Fig. 8, along with the fact that $f_0'/f_0 = 6.45$, we obtain $P'_{\text{out}}/P_{\text{avail}} = 3.48$ or a nominal gain of 5.3 db. By (25), it is found that the sideband resonator should be adjusted to couple a resistance of $R_{b0}' = 9.95$ ohms into the diode circuit at the upper-sideband frequency $f_0' = 6.45$ kMc.

Using $x_1^p = 430$ ohms, $w_p = wf_0/f_0^p = 0.0638$, and $R_g^p = R_s = 3.86$ ohms in (36), we get $(\Omega_1 l_1/r_g)_p = 7.1$ for the pump filter low-pass prototype. A Tchebycheff ripple of 0.5 db using an $n = 3$ resonator pump filter will be specified which by (38) calls for $d = 0.6265$. Then by (39), $e = 0.4855$. By Fig. 10, $U_3(0.6265) = 3.05$ and $U_3(0.4855) = 2.18$. By (40), $|\rho|_{\max} = 0.715$. By (41), a minimum power of $A(P_{\text{avail}})^p = 0.49\ (P_{\text{avail}})^p$ will be delivered to the diode where $(P_{\text{avail}})^p$ is the available pump power. Assuming that the diode is to be pumped with a peak-to-peak voltage of 6 v, it is estimated that the diode will absorb about 30 mw of pump power when $f^p = f_0^p = 5.45$ kMc. Thus, the required available pump power will be about $(P_{\text{avail}})^p = 30/A = 30/0.49 = 61$ mw, neglecting dissipation loss in the pump input filter. A 1-db dissipation loss would raise this figure to about 77 mw required available power.

[15] In computing R_g^p and x_1^p, the denominator in (16b) was neglected and the representation in Fig. 6(a) was used. This should cause little error in the performance estimates about to be computed since, as has been mentioned in Section IV, the denominator of (16b) will be varying quite slowly. It can therefore be regarded as constant within the frequency range of interest. As a constant, it would decrease the size of both R_g^p and x_1^p proportionally (about 25 per cent in this case), but would cancel out in the computation given by (36).

Let us now make analogous approximate calculations as described in Section VII using the same diode and diode circuit having $f_0 = 1$ kMc, $f_0^p = 5.45$ kMc, $x_1 = 145$ ohms, $x_1^p = 430$ ohms, etc., as before. Again, $w = 0.348$ will be used (half-octave input tuning range). In this case, the output will be taken at the lower-sideband frequency: $f_0' = f_0^p - f_0 = 5.45 - 1 = 4.45$ kMc. Since this gives negative resistance operation which increases the sensitivity to input impedance variations, the $n = 3$ resonator input filter will be assumed to have its Tchebycheff ripple reduced to 0.25 db which calls for $(\Omega_1 l_1/r_g) = 1.3034$.

The calculations as outlined in Section VII call for $(R_b)_{\text{nom}} = R_{b0} = 38.7$ ohms, $R_{b0}' = 8.0$ ohms (when $D = 2$). Assuming a stability parameter of $D = 2$, the nominal power gain would be 13.4 db, 8.1 db greater than for corresponding upper-sideband design. The pump power required should be the same in both cases.

Similar approximate calculations for a one-octave-tuning-range, lower-sideband up-converter were made, again assuming the same diode circuit and the same $f_0, f_0^p, f_0' = 4.45$ kMc values. A value of 0.25-db Tchebycheff ripple was assumed for the $n = 3$ resonator input filter. Since stability might be more of a problem using an octave input bandwidth, $D = 3$ was used. In this case, $(R_b)_{\text{nom}} = 78.8$, $R_{b0}' = 5.45$, the nominal gain is 8.8 db, $A = 0.273$, and the required available pump power is about $(P_{\text{avail}})^p = 30/0.273 = 110$ mw, neglecting pump filter dissipation loss and assuming that the diode needs 30 mw of absorbed pump power. If the pump filter has 1-db dissipation loss, the required available power would be about 138 mw.

IX. Conclusions

Properly designed devices of this type using voltage-tunable pump oscillators should have tuning ranges of the order of a half-octave to an octave, fast tuning capability, a useful amount of gain, no image response, and a low noise figure. Though some reflection loss in the pump circuit will be unavoidable, it appears that in many cases existing voltage-tunable oscillators will have adequate incident power.